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A κ gauge fixed type IIB superstring action on $AdS_5 \times S_5$

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ABSTRACT: We describe the type IIB supergravity background on $AdS_5 \times S_5$using the potentials of $AdS_{5|4} \otimes S_5$ and we use the supersolvable algebra associated to $AdS_{5|4}$ to compute the κ gauge fixed type IIB string action.

KEYWORDS: Superstrings and Heterotic Strings, Extended Supersymmetry.

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Contents

1. Introduction

It has become clear in the last period that in order to exploit the celebrated conjecture on the AdS/SYM [1, 2, 3] correspondence fully is of the utmost importance to be able to compute string amplitudes on $AdS_5 \times S_5$ background [11, 13]: this would be equivalent to control $N = 4$ SYM in the large N limit for all the possible values of the 't Hooft coupling constant $\lambda = g_{YM}^2 N$.

The problem has been tackled constructing the σ-model associated with the supergroup $\frac{SU(2,2|4)}{SO(1,4)\otimes O(5)}$ [4, 5] which is the global symmetry of the background; in order to be able to use the resulting action one has first of all to fix the κ-symmetry and until very recently [6] there was no proposal on how to do it.

It is purpose of this letter to obtain such a κ gauge fixed action but in order to achieve this we will use a little different approach and we will follow the supersolvable approach developed in [8].

This approach consists in the following steps:

1. find the field strengths and the reonomic parameterization corresponding to the $AdS_5 \times S_5$ background;

2. write the IIB superspace background in terms of the potentials of $AdS_{5|4} \otimes S_5 \equiv \frac{SU(2,2|4)}{SO(1,4)\otimes U(4)} \otimes \frac{SO(6)}{SO(5)}$ (which allows moreover to easily generalize to more general backgrounds);

3. find the supersolvable algebra associated with $AdS_{5|4}$. In doing so we project out $\frac{1}{2}$ of the fermionic coordinates thus fixing a gauge for the κ symmetry;

4. insert the previous parameterization in the appropriate action which , in this case, is the one computed by Grisaru, Howe, Mezincescu, Nilsson and Townsend [9].

We will now sketch these steps and give the results of each of them leaving for a future publication a more complete explanation along with the application to the $D3$ case [10].

2. Step 1: determining the superbackground

Using the notation of [7] [1] it is an easy matter to find the field strengths associated with the $AdS_5 \times S_5$ solution:

$$R^{ab}_{\ ..\,cd} = -e^2\,\delta^{ab}_{cd} \qquad\qquad R^{ij}_{\ ..\,kl} = e^2\,\delta^{ij}_{kl} \tag{2.1}$$

$$F_{abcde} = \frac{e}{5}\epsilon_{abcde} \qquad\qquad F_{ijklm} = \frac{e}{5}\epsilon_{ijklm} \tag{2.2}$$

$$F^{\alpha}_{\widehat{a}\widehat{b}\widehat{c}} = 0 \tag{2.3}$$

$$P_{\widehat{a}} = 0 \tag{2.4}$$

$$D_{\widehat{a}}\lambda = 0 \tag{2.5}$$

$$\rho_{\widehat{a}\widehat{b}} = 0\,, \tag{2.6}$$

where $\widehat{a}, \widehat{b}, \ldots = 0\ldots 9$, $a, b, \ldots = 0\ldots 4$ and $i, j, \ldots = 5\ldots 9$.

3. Step 2: expressing the superbackground through $AdS_{5|4} \otimes S_5$ generalized potentials

We want now to express the IIB generalized potentials $\left\{\omega^{\widehat{a}\widehat{b}}, V^{\widehat{a}}, \Psi, V^{\pm}_{\alpha}A^{\alpha}, B\right\}$, thought as fields living on the $AdS_{5|4}\otimes S_5$ superspace, using a linear combination of the following fields:

- the generalized potentials describing the 5D super anti de Sitter $AdS_{5|4}$ supergeometry 4 $\left\{\omega^{ab}(x,\Theta), P^a(x,\Theta), A^N_M(x,\Theta), B(x,\Theta), \psi_N(x,\Theta)\right\}$ with $(N = 1, \ldots, 4)$;
- the funfbein and spin connection $\{E^i(z), \omega^{ij}(z)\}$ describing the classical geometry of $S_5 = \frac{SO(6)}{SO(5)}$.

In order to proceed and enable the reader to reproduce the computations we need define in a more precise manner the previous mentioned fields:

[1] With the minor modifications given by $\psi \to \Psi$, $\psi^* \to \Psi_c \equiv \widehat{C}\overline{\Psi}^T$ and $\varpi^{ab} \to \Omega^{\widehat{a}\widehat{b}}$.

- the generalized potentials of the 5D super anti de Sitter superspace $AdS_{5|4}$ $\left\{\omega^{ab}(x,\Theta), P^a(x,\Theta), A^N_M(x,\Theta), B(x,\Theta), \psi_N(x,\Theta)\right\}$ satisfy the $su(2,2|4)$ Maurer-Cartan equations:

$$d\varpi^{ab} - \varpi^{ac}\varpi_c^{\ .b} + e^2 P^a P^b + \frac{e}{2}\overline{\psi}^N \gamma^{ab}\psi_N = 0\,, \tag{3.1}$$

$$dP^a - \varpi^a_{\ .b}P^b - \frac{i}{2}\overline{\psi}^N \gamma^a \psi_N = 0\,, \tag{3.2}$$

$$dB + \frac{e}{2}\overline{\psi}^N \psi_N = 0\,, \tag{3.3}$$

$$dA^N_{\ .M} + eA^N_{\ .P}A^P_{\ .M} + \overline{\psi}^N \psi_M = 0\,, \tag{3.4}$$

$$d\psi_M - \frac{1}{4}\varpi^{ab}\gamma_{ab}\psi_M + \frac{ie}{2}P^a\gamma_a\psi_M + e\psi_N A^N_{\ .M} = 0\,, \tag{3.5}$$

where $\overline{\psi} = \psi^\dagger\gamma_0$, $\psi_c = C\overline{\psi}^T$ and depend on $\{x^a, \Theta_N\}$ [2] $(N = 1, \ldots, 4)$ where the fermionic coordinates $\Theta_N = \begin{pmatrix} \theta_N \\ \xi_N \end{pmatrix}$ transform as spinors *only* with respect to $SO(1,4)$ and *not* with respect to $SO(1,4) \otimes SO(5)$: they are a set of 4 (symplectic) complex spinors in 5D which are the result of the dimensional reduction of two Majorana-Weyl 10D spinors. The Maurer-Cartan equations (3.1)-(3.5) are derived from the generic 1-form valued Lie algebra element

$$\mu = \begin{pmatrix} -\omega^{ab}\frac{1}{4}\gamma_{ab} + eP^a\frac{i}{2}\gamma_a & \sqrt{e}\,\psi_M \\ \sqrt{e}\,\overline{\psi}^N & A^N_{\ .M} \end{pmatrix} + B\,1_8\,, \tag{3.6}$$

which is defined by the condition

$$\mu^\dagger G + G\mu = 0 \quad G = \begin{pmatrix} -\gamma_0 & \\ & 1_N \end{pmatrix}. \tag{3.7}$$

This condition allows also to write the generic element of the $su(2,2|4)$ superalgebra as

$$g = \begin{pmatrix} t^{ab}\frac{1}{4}\gamma_{ab} + t^a\frac{i}{2}\gamma_a & \Theta_M \\ \overline{\Theta}^N & t^N_M \end{pmatrix} \tag{3.8}$$

and write the generic element of vector space associated with the coset $AdS_{5|4}$ either in the usual way as

$$l = \begin{pmatrix} t^a\frac{i}{2}\gamma_a & \Theta_M \\ \overline{\Theta}^N & 0 \end{pmatrix} \tag{3.9}$$

or using the solvable algebra associated with AdS_5 as

$$l = \begin{pmatrix} t^4\frac{i}{2}\gamma_4 + t^p\frac{i}{2}\gamma_p\frac{1+i\gamma_4}{2} & \sqrt{e}\,\Theta_M \\ \sqrt{e}\,\overline{\Theta}^N & 0 \end{pmatrix} \tag{3.10}$$

and compute a coset representative of the superspace exponentiating the previous algebra element.

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- the classical funfbein and spin connection $\{E^i(z), \omega^{ij}(z)\}$ of $S_5 = \frac{SO(6)}{SO(5)}$ endowed with the killing induced metric, i.e. negative definite, satisfy

$$dE^i - \varpi^i_{\ j} E^j = 0\,, \tag{3.11}$$

$$d\varpi^{ij} - \varpi^{ik}\varpi_k^{\ j} = e^2 E^i E^j \tag{3.12}$$

and depend on the coordinate z. Explicitly we have $(z^2 = \eta_{ij} z^i z^j = -\delta_{ij} z^i z^j)$[3]

$$E^i = \frac{2}{e}\frac{dz^i}{1-z^2}\,, \tag{3.13}$$

$$\varpi^{ij} = \frac{4z^{[i}dz^{j]}}{1-z^2}\,. \tag{3.14}$$

After having properly defined the fields we are going to use we can proceed in a similar way as done in [8] for the 11D supergravity and express the IIB generalized potentials through the superspace potentials; we obtain

$$V^a = P^a(x,\Theta) \qquad V^i = E^i(z) + \frac{1}{2}\overline{\eta}_N \tau^i \eta^M\, A^N_M(x,\Theta)\,, \tag{3.15}$$

$$\Omega^{ab} = \varpi^{ab}(x,\Theta) \qquad \Omega^{ij} = \varpi^{ij}(z) - \frac{e}{2}\overline{\eta}_N \tau^{ij}\eta^M\, A^N_M(x,\Theta)\,, \tag{3.16}$$

$$\Omega^{ai} = 0\,, \tag{3.17}$$

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0_{16\times 1}\,, \\ \psi_N(x,\Theta)\otimes \eta^N(z)\end{pmatrix}\,, \tag{3.18}$$

$$\epsilon_{\alpha\beta} V^\alpha_+ A^\beta = -\frac{i}{e}\overline{\psi}_{cN}\psi_M\, \overline{\eta}^N_c \eta^M\,, \tag{3.19}$$

$$\begin{pmatrix} V^1_- & V^2_-\,, \\ V^1_- & V^2_+ \end{pmatrix} = const \in SU(1,1)\,, \tag{3.20}$$

where we have introduced the 4 S_5c-number Killing spinors ($N = 1\ldots 4$) defined by

$$D_{SO(5)}\eta^N \equiv \left(d - \frac{1}{4}\varpi^{ij}\tau_{ij}\right)\eta^N = +\frac{e}{2}\tau_i \eta^N E^i \tag{3.21}$$

and their conjugates defined as $\eta_{cN} = C_5 \overline{\eta}_N$ with $\overline{\eta} \equiv \eta^\dagger$ with the normalization $\overline{\eta}_N \eta^M = \delta^M_N$.

The proof is based on application of the standard Fierz identities.

4. Step 3: fixing the κ symmetry using the supersolvable algebra

We are now ready to perform the third step: to compute the solvable superalgebra associated with the anti de Sitter superspace $AdS_{5|4}$, thus fixing the κ symmetry (see

[3] The general superalgebra $su(2,2|M)$ is defined by the Maurer-Cartan system (3.1)-(3.5) with the index N assuming values in $\{1,\ldots,M\}$ where all potentials depend on the coordinates $\{x^{ab}, x^\bullet, x^a, \Theta_N\}$ $(a,b,\ldots = 0,\ldots,4)$ of the associated supergroup. The $AdS_{5|M}$ potentials satisfy the same equations but depend only on the coordinates of the associated coset, i.e. $\{x^a, \Theta_N\}$.

appendix C for a complete discussion of the reasons why this works properly). The resulting solvable superalgebra is described by the Maurer-Cartan equations:

$$dP = 0\,, \tag{4.1}$$

$$dN^p + eN^pP - i\frac{e}{\sqrt{2}}\overline{\chi}^N\gamma^p\chi_N = 0\,, \tag{4.2}$$

$$d\chi_N - \frac{e}{2}P\chi_N = 0\,, \tag{4.3}$$

where $p, q, \ldots \in \{0,1,2,3\}$, $P \equiv P^4$, $N^p \equiv \frac{\varpi^{p4}+e\,P^p}{\sqrt{2}}$ and $\chi_N = \frac{1+i\gamma^4}{2}\psi_N$ with all the other linearly independent $AdS_{5|4}$ potentials set to zero. From the knowledge of which 1-forms survive in (3.6) and from the explicit expression for the generic element of the $su(2,2|4)$ algebra (3.8) we can write the generic element of the supersolvable algebra as

$$g(u, y^p, \theta) = \begin{pmatrix} -\frac{e}{2}u\,1_2 & \frac{i}{\sqrt{2}}y^p\,\sigma_p & \sqrt{e}\,\theta_N\,, \\ . & \frac{e}{2}u\,1_2 & . \\ . & \sqrt{e}\,\vartheta^{\dagger N} & . \end{pmatrix} = u\mathcal{C} + y^p\mathcal{N}_p + \theta^{\dagger N}Q_N + \overline{Q}^N\theta_N\,. \tag{4.4}$$

Then we can obtain an explicit expression for the potentials by using the usual expression $\mu = G^{-1}dG$ with $G = \exp(u\mathcal{C})\,\exp(y^p\mathcal{N}_p)\,\exp(\theta^{\dagger N}Q_N + \overline{Q}^N\theta_N)$

$$P = du\,, \tag{4.5}$$

$$N^p = dy^p - ey^pdu + i\frac{e}{2\sqrt{2}}\left(\vartheta^{\dagger N}\tilde{\sigma}^p d\theta_N - d\vartheta^{\dagger N}\tilde{\sigma}^p\theta_N\right)\,, \tag{4.6}$$

$$\chi_N = \begin{pmatrix} d\theta_N - \frac{e}{2}\theta_N du \\ 0 \end{pmatrix}\,, \tag{4.7}$$

$$\overline{\chi}^N = \begin{pmatrix} 0 & d\vartheta^{\dagger N} - \frac{e}{2}\vartheta^{\dagger N}du \end{pmatrix}\,. \tag{4.8}$$

From these expression (4.5)-(4.8) we extract the κ gauge-fixed expression of the $AdS_{5|4}$ potentials as

$$P^p = \frac{1}{e}\varpi^{p4} = \frac{1}{e}\left(\rho dx^p + \frac{ie}{4}\rho\left(\vartheta^{\dagger N}\tilde{\sigma}^p d\theta_N - d\vartheta^{\dagger N}\tilde{\sigma}^p\theta_N\right)\right)\,, \tag{4.9}$$

$$P^4 = \frac{1}{e}\frac{d\rho}{\rho}\,, \tag{4.10}$$

$$\psi_N = \sqrt{\rho}\begin{pmatrix} d\theta_N \\ 0 \end{pmatrix}\,, \tag{4.11}$$

$$A^N_{\;.M} = B = 0\,, \tag{4.12}$$

where we have changed variables as $u \to \rho = e^{eu}$, $y^p \to x^p = \frac{1}{\sqrt{2}}e^{-eu}y^p$, $\theta \to \theta' = e^{-eu/2}\theta$ and dropped the prime for the new θ variable.

One could wonder why this procedure fixes a gauge for the κ symmetry. The reason is very simple: it is equivalent to give a projector for the fermionic coordinates. In fact to fix the κ symmetry is roughly equivalent to throw away half of the fermionic

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coordinates and then to set these coordinates to zero in the general expression for the $AdS_{5|4}$ potentials. All these steps are accomplished in one step by computing the potentials for the supersolvable algebra but we could as well have performed all these steps explicitly by first computing the $AdS_{5|4}$ potentials and hence the background fields as a function of $\left\{ x, z, \Theta_N = \begin{pmatrix} \theta_N \\ \xi_N \end{pmatrix} \right\}$ and then setting $\xi_N = 0$ which are the fermionic coordinates projected out by the supersolvable gauge. What one has to check is whether the projector on the Θ_N coordinates is compatible with the κ symmetry variation and with the equations of motion: this is done in app. C where we show that with this gauge fixing we are able to describe all string excitations but the ones which do not satisfy

$$\Pi_\tau^p \Pi_{p\tau} + \Pi_\sigma^4 \Pi_{4\sigma} + \Pi_\sigma^i \Pi_{i\sigma} \neq 0 \,, \tag{4.13}$$

where $V^{\widehat{a}} = \Pi_\tau^{\widehat{a}}\, d\tau + \Pi_\sigma^{\widehat{a}}\, d\sigma$. This kind of impossibility of describing all the string excitations is similar to what happens in the usual GS superstring in the light cone gauge on flat superspace where we can describe all excitations but the massless ones propagating in the $+$ direction, the difference is that here we cannot easily understand which states do not satisfy eq. (4.13); we can nevertheless find a class of such states which is made by massless states with $\Theta = 0$ propagating at $r = const$ since in this case we have trivially $\Pi_\sigma^4 = \Pi_\sigma^i = 0$ and $\Pi_\tau^p \Pi_{p\tau} = 0$.

5. The action

We have now all the necessary expressions which can be inserted into the action given appendix B (with $\phi = \frac{\pi}{2},\ c_1 = 1$) or, that is the same, in the general action given in [9] (after a careful comparison with [12][4] too) and get[5]

---

[4]The dictionary between the notations used by Howe and West [12] and ours [7] is obtained setting $B = 1$ and $\widehat{C} = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$ and it is given by

$$\gamma_0 = \Gamma_0 \qquad \gamma_{1,\dots,9} = -\Gamma_{1,\dots,9}$$
$$\Omega_a^{\cdot b} = \Omega^{\widehat{b}}_{\cdot \widehat{a}}$$
$$E^a = V^{\widehat{a}} \qquad \begin{pmatrix} E^\alpha \\ 0 \end{pmatrix} = \Psi\, e^{i\pi/4}$$
$$P_{HW} = P \qquad Q_{HW} = -\tfrac{1}{2} Q \qquad \begin{pmatrix} 0 \\ \Lambda_\alpha \end{pmatrix} = \frac{1}{2} \lambda\, e^{-i\pi/4}$$
$$F_{HW} = \tfrac{i}{4} \epsilon_{\alpha\beta} V_+^\alpha dA^\beta \quad .$$

Notice that for this comparison we have used $\Gamma_{11} = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$ instead of the more usual expression $\Gamma_{11} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ which is the one used in the paper.

[5]In the main formula (19) of [9] the factor $\frac{1}{2}$ should be moved from the second addend to the first. This can be seen either by comparing with the expression for the flat spacetime (1) or by looking at the expression for the variation of the action (23).

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$$
\begin{aligned}
S = \int d^2\xi\, \sqrt{-g}\, g^{\alpha\beta}\frac{1}{2} \times \\
&\times \left\{ \eta_{pq}\frac{\rho^2}{e^2}\left[\partial_\alpha x^p + \frac{ie}{4}\left(\vartheta^{\dagger N}\tilde{\sigma}^p\partial_\alpha\theta_N - \partial_\alpha\vartheta^{\dagger N}\tilde{\sigma}^p\theta_N\right)\right]\times \right.\\
&\times \left.\left[\partial_\beta x^q + \frac{ie}{4}\left(\vartheta^{\dagger M}\tilde{\sigma}^q\partial_\beta\theta_M - \partial_\beta\vartheta^{\dagger M}\tilde{\sigma}^q\theta_M\right)\right] - \frac{1}{e^2}\frac{\partial_\alpha\rho\partial_\beta\rho}{\rho^2} - \frac{4}{e^2}\delta_{ij}\frac{\partial_\alpha z^i\partial_\beta z^j}{(1-z^2)^2}\right\} - \\
&- \frac{i}{4e}\rho\,\left[d\theta^{\dagger N}\sigma_2 d\theta^{*M}\,\overline{\eta}_N(z)\eta_{cM}(z) + d\theta^T_N\sigma_2 d\theta_M\,\overline{\eta}^N_c(z)\eta^M(z)\right]. \qquad (5.1)
\end{aligned}
$$

This action is highly non linear and in order to be able to compute the string scattering amplitudes we have now to solve the hardest part of the problem, i.e. to find a way to quantize this expression, perhaps rewriting the previous action in term of free fields.

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Note Added.

By request of the referee I have added appendices B and C where it is explained why and in which way the κ symmetry is fixed by this approach.

A. Conventions

- Indices: $\hat{a},\hat{b},\ldots \in \{0,\ldots,9\}$; $a,b,\ldots \in \{0,1,2,3,4\}$, $i,j,\ldots \in \{5,\ldots,9\}$; $p,q,\ldots \in \{0,1,2,3\}$, $t,u,v,\ldots \in \{4,\ldots,9\}$; $\alpha,\beta,\ldots \in \{0,1\}$
- Metrics: $\eta_{\hat{a}\hat{b}} = diag(+1,-1,\ldots-1)$, $\eta_{ab} = diag(+1,-1,-1,-1,-1)$, $\eta_{ij} = diag(-1,-1,-1,-1,-1)$; $\eta_{\alpha\beta} = diag(+1,-1)$
- Epsilons: $\epsilon_{0\ldots 9} = \epsilon_{0\ldots 4} = \epsilon_{5\ldots 9} = 1$; $\epsilon_{01} = 1$
- 1+9D gamma matrices

$$
\{\Gamma_{\hat{a}},\Gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}}
$$
$$
\Gamma_{11} = \Gamma_0\ldots\Gamma_9 = \begin{pmatrix} 1_{16} & \\ & -1_{16}\end{pmatrix}
$$
$$
\Gamma^T_{\hat{a}} = -\hat{C}^{-1}\,\Gamma_{\hat{a}}\,\hat{C} \quad \hat{C}^T = -\hat{C} \quad \hat{C}^\dagger = \hat{C}^{-1}
$$

- 1+4D (AdS_5) gamma matrices

$$
\{\gamma_a,\gamma_b\} = 2\eta_{ab}
$$
$$
\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4 = 1_4
$$
$$
\gamma^T_a = +C^{-1}\,\gamma_a\,C \quad C^T = -C \quad C^\dagger = C^{-1}\,.
$$

Moreover we use the following (1+4)D γ explicit representation:

$$\gamma_p = \begin{pmatrix} & \sigma_p \\ \widetilde{\sigma_p} & \end{pmatrix} \; \gamma_4 = \begin{pmatrix} i\,1_2 & \\ & -i\,1_2 \end{pmatrix} \; C = \begin{pmatrix} i\sigma_2 & \\ & i\sigma_2 \end{pmatrix}$$

with $p, q, \ldots = 0 \ldots 3$, $\sigma_p = \{1_2, -\sigma_1, -\sigma_2, -\sigma_3\}$ and $\widetilde{\sigma}_p = \{1_2, \sigma_1, \sigma_2, \sigma_3\}$

- 0+5D (S_5) gamma matrices

$$\begin{aligned} \{\tau_i, \tau_j\} &= 2\eta_{ij} \\ \tau_5\tau_6\tau_7\tau_8\tau_9 &= i\,1_2 \\ \tau_i^T &= +C^{-1}\,\tau_i\,C \qquad C_5 = C \end{aligned}$$

- 1+9D gamma expressed through AdS_5 γ^a and S_5 τ^i gammas

$$\begin{aligned} \Gamma^{\widehat{a}} &= \left\{\gamma^a \otimes 1_4 \otimes \sigma_1 \,,\, 1_4 \otimes \tau^i \otimes (-\sigma_2)\right\} \\ \widehat{C} &= C \otimes C_5 \otimes \sigma_2 . \end{aligned}$$

B. The superstring action on $AdS_5 \otimes S_5$ in first order formalism and its κ symmetry.

It is purpose of this appendix to determine the form of the superstring action in first order formalism and its κ symmetry: this is done at the same time starting from the ansatz inspired by the knowledge of the second order action ([9]) and from the fact that the dilatons are constant:

$$\begin{aligned} S = \int_\Sigma & c_1 \Pi^{\widehat{a}}_\alpha \, i_* V^{\widehat{b}} \wedge e^\beta \, \eta_{\widehat{a}\widehat{b}}\, \epsilon_{\alpha\beta} + c_2 \Pi^{\widehat{a}}_\alpha \Pi^{\widehat{b}}_\beta \, \eta_{\widehat{a}\widehat{b}}\, \eta^{\alpha\beta}\, \epsilon_{\gamma\delta}\, e^\gamma \wedge e^\delta + \\ & + c_3 V_+ * A - c_3^* V_- * A \,, \end{aligned} \tag{B.1}$$

where $V_\pm * A = \epsilon_{\alpha\beta} V_\pm^\alpha A^\beta$, $\alpha, \beta, \ldots \in \{0, 1\}$ are worldsheet indices and i_* is the pullback on the string worldsheet Σ due to the superimmersion $i:\ \Sigma \ \rightarrow\ AdS_5 \otimes S_5$.

The relative coefficient of the first two terms of the first line in eq. (B.1) is determined to be

$$c_2 = -\frac{1}{4} c_1 \tag{B.2}$$

by the request that the $\Pi^{\widehat{a}}_\alpha$ equation of motion yields

$$\frac{\delta S}{\delta \Pi} = 0 \Longrightarrow i_* V^{\widehat{a}} = \Pi^{\widehat{a}}_\alpha e^\alpha \,. \tag{B.3}$$

The zweibein equation of motion gives the Virasoro constraints:

$$\frac{\delta S}{\delta e} = 0 \Longrightarrow \Pi_\alpha \bullet \Pi_\beta = \frac{1}{2} \eta_{\alpha\beta} \Pi^2 \,. \tag{B.4}$$

From the request of the action (B.1) (in the 1.5 formalism) to have a κ symmetry we determine the relative value of the coefficients of last two terms in eq. (B.1) with respect the first term in the first line:

$$c_3 = \frac{1}{4} e^{i\phi} c_1 \quad \forall \phi \tag{B.5}$$

and the vector field $\overrightarrow{\epsilon} = \delta_\kappa \Theta_N \overrightarrow{\partial_{\Theta_N}} + \delta_\kappa x \overrightarrow{\partial_x}$ associated to the κ symmetry and the variation of the zweibein:

$$\delta_\kappa e^\alpha = i \overline{\kappa}^{N\alpha} \psi_N - i \overline{\psi}^N \kappa_N^\alpha .$$

The vector field $\overrightarrow{\epsilon}$ is defined by the properties

$$\overrightarrow{\epsilon} \mid \psi_N = \epsilon_N = \Pi_\alpha^a \, \gamma_a \kappa_N^\alpha - i \Pi_\alpha^i \, J_i|_N^P \, \kappa_P^\alpha , \qquad \overrightarrow{\epsilon} \mid V^i = 0 , \tag{B.6}$$

$$\kappa_{c\alpha}^N = -i e^{i\phi} \, L^{NP} \, \epsilon_{\alpha\beta} \, \kappa_{P\beta} \tag{B.7}$$

and its action by mean of the Lie derivative gives the supersymmetric variations (up to local Lorentz and gauge transformations) of the other fields

$$\delta_\kappa V^a = \frac{i}{2} \left(\overline{\epsilon}^N \gamma^a \psi_N - \overline{\epsilon}_{cN} \gamma^a \psi_c^N \right) , \tag{B.8}$$

$$\delta_\kappa V^i = \frac{1}{2} \, J^i|_N^M \, \left(-\overline{\epsilon}^N \psi_M + \overline{\epsilon}_{cM} \psi_c^N \right) , \tag{B.9}$$

$$\delta_\kappa (V_+ * A) = 2 \, \overline{\epsilon}_{cN} \gamma^a \psi_M \, L^{NM} \, V_a + 2i \, \overline{\epsilon}_{cN} \psi_M \, J^i|^{NM} \, V_i , \tag{B.10}$$

where we have defined

$$J^i|_N^M = \overline{\eta}_N \tau^i \eta^M = \overline{\eta}_c^M \tau^i \eta_{cN} \tag{B.11}$$

$$J^i|^{MN} = \overline{\eta}_c^N \tau^i \eta^M = -J^i|^{NM} , \tag{B.12}$$

$$J^i|_{MN} = \overline{\eta}_M \tau^i \eta_{cN} = -J^i|_{NM} , \tag{B.13}$$

$$L^{MN} = \overline{\eta}_c^M \eta^N = -L^{NM} , \tag{B.14}$$

$$L_{MN} = \overline{\eta}_M \eta_{cN} = -L_{NM} . \tag{B.15}$$

It is important in proving that the action (B.1) is invariant under the κ symmetry variations given by eqs. (B.6), (B.8)-(B.10) to use the following Fierz identities:

$$J^i|_N^M \, L^{NP} = L^{MP} \, J^i|_P^N = J^i|^{MN} , \tag{B.16}$$

$$L^{MN} \, L_{NP} = \delta_P^M , \tag{B.17}$$

$$J^i|_N^M \, J^j|_P^N + J^j|_N^M \, J^i|_P^N = 2\eta^{ij} \, \delta_P^M . \tag{B.18}$$

The derivation of these Fierz identities is based on the fact that η^N are a basis of the vector space on which the spinor irrep acts, i.e. $\overline{\eta}_N \tau \, \eta^N \propto tr(\tau)$.

Before closing this appendix we would like to make some comments:

- here we have 4 complex $\kappa_{N\alpha}$ parameters instead of the usual real two because our superspace is five dimensional and the ψ_N (Θ_N) are the dimensional reduction of the usual 10D Majorana Ψ (Θ) hence they transform as spinors under $SO(1,4)$ *only*;

- Eq. (B.7) is the analogous of the (anti)selfduality condition imposed on the κ_α parameter in the usual GS superstring;

- even if it is not immediate to see, the ψ_N contraction involves a projector applied on the κ parameters. We can in fact define

$$\mathcal{P}^{(\pm)}_\alpha|^M_N = \Pi^a_\alpha\,\gamma_a\,\delta^M_N \pm i\,\Pi^i_\alpha\,J^i|^M_N \tag{B.19}$$

and verify the following properties:

$$\begin{aligned}\mathcal{P}^{(+)}_\alpha|^M_N + \mathcal{P}^{(-)}_\alpha|^M_N &= 2\Pi^a_\alpha\,\gamma_a\,\delta^M_N\,,\\ \mathcal{P}^{(-)}_\pm|^M_N\mathcal{P}^{(+)}_\pm|^N_P &= 0\,,\\ \mathcal{P}^{(s)}_\pm|^M_N\mathcal{P}^{(s)}_\pm|^N_P &= 2\Pi^a_\pm\,\gamma_a\,\mathcal{P}^{(s)}_\pm|^M_P\,,\end{aligned}$$

which shows as the $\mathcal{P}$s be a projectors on the space where $\Pi^a_\pm\,\gamma_a$ is invertible, i.e. when $\Pi^a_\pm\,\eta_{ab}\Pi^b_\pm \neq 0$. Moreover we have

$$\epsilon_N \pm ie^{-i\phi}L_{NP}\epsilon^P_c = 2\mathcal{P}^{(-)}_\pm|^M_N\kappa_{M\mp}\,, \tag{B.20}$$

which clearly shows that at least half the degrees of freedom associated with κ_N are projected out.

C. How the supersolvable algebra fixes the κ symmetry.

In order to discuss and explain the issue of why the κ symmetry is properly fixed which by the gauge which is automatically chosen by the supersolvable algebra we can either work using the general formulae derived in second order formalism in ([9]) or use the formulae written in first order formalism in the previous app. B: in both cases where we can easily find the gravitino variations but there is a big difference between the two results since while the formulae in ref. [9] refer to the 10D gravitini Ψ the ones in the previous appendix refer to the dimensional reduced ones ψ_N with which we have been working in this paper. We will hence follow this second approach.

As a first step we write down the supersolvable projector associated with the fermionic coordinates which are projected out:

$$P = \frac{1 + i\gamma_4}{2}$$

and we notice that the same projector P has to be used to find the gravitini components which are projected out. In the following we find under which conditions we can find a vector field $\overrightarrow{\epsilon}$ which can be used to eliminate the wanted components of the gravitini ψ_N .

To this purpose we write down the contraction of the vector field $\overrightarrow{\epsilon}$ generating the κ symmetry on $P\psi_N = \psi^{(+)}_N$ and $(P\psi_N)_c = Q\psi^N_c = \psi^{N(-)}_c$ where $Q = \frac{1-i\gamma_4}{2}$. After the elimination of κ_{N1} using eq. (B.7), eq. (B.6) becomes

$$\begin{aligned} \epsilon_N^{(+)} &= \mathcal{A}_0|_N^M \,\lambda_M + ie^{-i\phi}\mathcal{C}_1|_N^M \,L_{MP}\,\lambda_c^P + \\ &\quad +\mathcal{C}_0|_N^M \,\mu_M + ie^{-i\phi}\mathcal{A}_1|_N^M \,L_{MP}\,\mu_c^P\,, \qquad \text{(C.1)} \\ ie^{-i\phi}L_{NP}\,\epsilon_c^{P(-)} &= \mathcal{A}_1|_N^M \,\lambda_M + ie^{-i\phi}\mathcal{C}_0|_N^M \,L_{MP}\,\lambda_c^P + \\ &\quad +\mathcal{C}_1|_N^M \,\mu_M + ie^{-i\phi}\mathcal{A}_0|_N^M \,L_{MP}\,\mu_c^P\,, \qquad \text{(C.2)} \end{aligned}$$

where we have defined $\mu_M = P\kappa_{M0}$, $\lambda_M = Q\kappa_{M0}$ and

$$\begin{aligned} \mathcal{A}_\alpha|_N^M &= \Pi_\alpha^p\,\gamma_p\,\delta_N^M\,, \\ \mathcal{C}_\alpha|_N^M &= \Pi_\alpha^4\,\gamma_4\,\delta_N^M - i\,\Pi_\alpha^i\,J^i|_N^M\,. \end{aligned}$$

We can now make the following linearly independent combinations:

$$\begin{aligned} \Sigma_N = \epsilon_N^{(+)} + ie^{-i\phi}L_{NP}\epsilon_c^{P(-)} &= (\mathcal{A}_0 + \mathcal{C}_1)\,|_N^M \left[\lambda_M + ie^{-i\phi}L_{MP}\lambda_c^P\right] + \\ &\quad + (\mathcal{A}_1 + \mathcal{C}_0)\,|_N^M \left[\mu_M + ie^{-i\phi}L_{MP}\mu_c^P\right]\,, \qquad \text{(C.3)} \\ \Delta_N = \epsilon_N^{(+)} - ie^{-i\phi}L_{NP}\epsilon_c^{P(-)} &= (\mathcal{A}_0 - \mathcal{C}_1)\,|_N^M \left[\lambda_M - ie^{-i\phi}L_{MP}\lambda_c^P\right] - \\ &\quad - (\mathcal{A}_1 - \mathcal{C}_0)\,|_N^M \left[\mu_M - ie^{-i\phi}L_{MP}\mu_c^P\right]\,. \qquad \text{(C.4)} \end{aligned}$$

Since these combinations are linearly independent also on the complex field because under charge conjugation each of them transforms in its selves we can consider them independently and we can, for example, try to solve the Σ equation (C.3) for either $\left[\lambda_M + ie^{-i\phi}L_{MP}\lambda_c^P\right]$ or $\left[\mu_M + ie^{-i\phi}L_{MP}\mu_c^P\right]$.

We find that in both cases the necessary condition for the invertibility of the matrices $(\mathcal{A}_0 + \mathcal{C}_1)$ and $(\mathcal{A}_1 + \mathcal{C}_0)$ is (using the Virasoro constraint eq. (B.4)) that

$$\Pi_\tau^p\Pi_{p\tau} + \Pi_\sigma^4\Pi_{4\sigma} + \Pi_\sigma^i\Pi_{i\sigma} \neq 0\,. \qquad \text{(C.5)}$$

One could wonder whether this is a sufficient condition for the solvibility of eqs. (C.3), (C.4), the answer is yes since would the sum of the images of the two matrices $(\mathcal{A}_0 + \mathcal{C}_1)$ and $(\mathcal{A}_1 + \mathcal{C}_0)$ be the whole space spanned by Σ then it should be possible to find a number α such that $(\mathcal{A}_0 + \mathcal{C}_1) + \alpha\,(\mathcal{A}_1 + \mathcal{C}_0)$ is always invertible but it is easy to verify that the determinant of the previous combination is proportional to eq. (C.5) too.

We conclude therefore that states which do not satisfy eq. (C.5) cannot be described in this gauge.

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